3.21 Complex Numbers

Imaginary Number: $i=\sqrt{-1}$ and $i^2=-1$

Complex Numbers: The set of all numbers of the form $a+bi$ where a and b are real numbers.

a is called the **real part** and b is called the **imaginary part**. If $b\neq 0,$ then $a+bi$ is an **imaginary number.** The form $a+bi$ is called the **standard form** of a complex number.

Examples: Determine whether each complex number is real or imaginary and write it in standard form.

a) $4i$ b) $3-6i$ c) 5 d) $\dfrac{i-3\pi}{4}$

Addition, Subtraction, and Multiplication of Complex Numbers.

$(a+bi)+(c+di)=(a+c)+(b+d)i$

$(a+bi)-(c+di)=(a-c)+(b-d)i$

$(a+bi)(c+di)=(ac-bd)+(bc+ad)i$ or use FOIL.

Examples: Perform the indicated operations.

a) $(6+2i)+(4-3i)=$ b) $(7-4i)-(-2+8i)=$

c) $(6+5i)(8+3i)=$ d) $(1-i)(4+i)=$

Powers of *i*:

Since $i^2=-1,\quad i^3=i^2\cdot i=-1\cdot i=-i$, and $i^4=i^2\cdot i^2=-1\cdot -1=1$

The first eight powers are listed here:

$i^1=i$ $i^5=i$

$i^2=-1$ $i^6=-1$

$i^3=-i$ $i^7=-i$

$i^4=1$ $i^8=1$

The powers of *i* continue in this pattern.

Examples: Simplify the power of i.

a) $i^{35} =$ b) $i^{29} =$ c) $i^{98} =$ d) $i^{48} =$

Theorem: If a and b are real numbers, then the product of $a+bi$ and its conjugate $a-bi$ is the real number a^2+b^2. $(a+bi)(a-bi)=(a^2+b^2)$

Examples: Find the product of the complex number and its conjugate.

a) $3-7i$ b) $2+9i$ c) i

Examples: Write each quotient in the form $a+bi$.

a) $\dfrac{6-2i}{3} =$ b) $\dfrac{2}{8+9i} =$ c) $\dfrac{4-5i}{3+2i} =$

Roots of Negative Numbers

For any positive real number b, $\sqrt{-b} = i\sqrt{b}$.

Examples: Write each expression in the form $a+bi$, where a and b are real numbers.

a) $\sqrt{-5}+\sqrt{-8} =$ b) $\sqrt{-20}\left(\sqrt{-6}-\sqrt{-4}\right) =$

c) $\dfrac{-2+\sqrt{-48}}{2} =$

Example: Does the complex number $x = 1+3i\sqrt{2}$ satisfies the equation $x^2-2x+4=0$?